# Conceptual structure of natural numbers

## Modeling number cognition in conceptual spaces

### Description for the General Audience

People use numerals in different ways and in a variety of meanings: to determine how exactly many elements are in a collection, to estimate how approximately many they are, to scrupulously enumerate individual elements, to compute (*e.g.,* to add or to multiply), to write computer programs or to encrypt messages. Most cultures use a single system of numerals, such as Arabic numerals, to express all these meanings. This multiplicity of meanings of numerals is responsible for the complexity of the process of acquiring the number concept.

Research by cognitive psychologists shows that there are several basic cognitive abilities that influence how children learn to understand the number system. First, the ability to estimate (approximate) the number of elements in a given collection. Second, subitizing, which is the ability to determine the exact number of elements in small collections (up to four, sometimes five elements) without counting. We share these two abilities with animals. Third, the ability to pair elements from two collections ("one-to-one correspondence") to see which has more elements. Fourth, the ability to learn by heart successive names for numbers.

There are several different types of experimental research that allows assessing to what extent a person (child) has mastered a given ability. These abilities are acquired to some extent independently and it is sometimes argued that it is only by grasping the so-called "cardinality principle" – meaning that the size of a collection is named by the last numeral used to count its elements – enables to combine all these different abilities into a unified conceptual system. Some cognitive psychologists even believe that only when a child understands the cardinality principle will the concept of natural number be created.

My project aims to create a series of models for all the basic cognitive abilities involved in the formation of the natural number concept. These models will allow us to understand these abilities, and to trace the process by which they combine into a unified conceptual system.

The models I propose are based on the theory of conceptual spaces formulated by Peter Gärdenfors. This way of modelling differs from the computational models that are traditionally used to analyse the functioning of cognitive systems for processing quantitative, numerical and numerical information. Gärdenfors models have already been successfully used in many other fields to describe ranges of meaning and conceptual structures and learning concepts, especially in natural language semantics, but also to describe the dynamics of scientific theories.

In the course of our project, I will investigate to what extent the way children learn numerical concepts can be compared to the dynamics of the development of scientific theories (this approach was put forward by Alison Gopnik in her theory of child scientists). I will systematically compare the proposed models with the results of laboratory experiments conducted by cognitive psychologists specialised in the study of quantitative and numerical information processing. In particular, with those studies aimed at understanding the cognitive stages of number learning and the stages of formation of the concept of natural number.

The results of my research will provide a deeper understanding of the cognitive structures and processes underlying a fully developed conceptual system for numbers and numerical expressions. This will allow the formulation of new conclusions for experimental studies that can be performed to determine a child's level of mathematical development. Most importantly, the models will provide material for early education professionals who already rely on the same empirical research that inspired our project.