Popular science description of research for the project "Perturbations of locally Hamiltonian flows and generalized interval exchange transformations"

The study of dynamical systems was originated in the early twentieth century. They appeared in the centre of interest among scientist, as they mathematically model many phenomena occurring in physics, astronomy and economy, among other fields of science. They can for instance model the movement of celestial bodies, reflection of light-rays as well as the movement of particles. The *locally Hamiltonian flows*, that appear in the title of this project, serve exactly to describe those notions.

Many mathematical models originate as ”ideal” models, that is, models in which we take into account a vastly reduced number of factors. For instance, when we consider a movement of a gas particle inside the box, we typically ignore the influence of the gravitational forces between the considered particle and the particles that form the box. A natural way to make a model more ”real” is to add a disturbance that will correspond to forces we typically ignore in ideal models. These are the *perturbations* that we intend to consider.

The perturbed systems, by their nature, are more complicated to consider than simpler models. Since it is rather undesirable to work with a more intricate object, a natural question arises: was the perturbation really required, to study the object, that we are interested in? To be more specific, is there a different way to observe the perturbed object, so that without any loss of information, we can focus on investigating the ideal model? This change of the point of view, can be seen as an existence of *conjugacy* between a perturbed and unperturbed system. One of the main goals of our project, is to determine whether such conjugacy exists and, if it does, what are its properties.

Sometimes it is worth to reduce the problem to a situation, where we have less information, but the properties that we obtain on the reduced model still give a valuable impact on the study of the original problem. Imagine for example, that a ray of light travels through a large finite space with finitely many obstacles and from time to time is caught by a linear detector. We record only the position of the point that the ray passed through the linear detector. If, for example, the ray penetrates every corner of the whole space, we may safely assume that it would cover also the surface of the detector. We get valuable information by measuring on a much smaller scale. The mathematical objects that describe consecutive passings of the ray through the detector are *generalized interval exchange transformations (GIETs).*

Hence to study the perturbed locally Hamiltonian flow, we intend also to take a closer look at the GIETs. Firstly, to study the perturbations of the flows described earlier, we want to see how much we can simplify a GIET under consideration. That is, can we see it, by changing the measurement, as the transformation obtained by studying the unperturbed flow. Secondly, we want to understand how to measure the sets of points in the domain of our GIET, so that its action does not change this measure. Then, a classical ergodic result by Birkhoff, yields a statistical information on how the points behave in the long run under iterations of our system.