Networks have become a pervasive element of 21st century life. While real-world networks can be quite different, they have the same underlying structure: objects connected by links. Extracting this common feature leads to an abstract model that enables the study of properties shared by a variety of networks. A ‘graph’ consists of a set of objects called vertices, where pairs of vertices are joined by links called edges. When modeling transportation networks, the vertices may be cities and the edges roads, railway lines, or airline routes. Molecular chemists consider graphs whose vertices are atoms and whose edges are molecular bonds. Statistical physicists use graphs to model crystal formation. The world wide web defines a graph whose vertices are web pages and edges are hyperlinks. In fact, the connections of any physical or conceptual network can be modeled using a graph. Graph theory, which is the mathematics of networks, has sprung to prominence as a model of real-world processes, and has become a central topic in mathematics itself, as well as in applied disciplines, especially computer science.

In this project we study planar graphs, the graphs that can be drawn in the plane without edge crossings. Planar graphs are natural objects that have been studied for the last two centuries. One of the most famous results in mathematics, the Four Color Theorem, is about planar graphs: Every planar graph is 4-colorable. Proving such an innocent looking statement turned out to require a deep understanding of the structure of planar graphs, and was a main achievement in the second part of the 20th century.

Nowadays, one might think that everything about the structure of planar graphs that is worth knowing has already been discovered. This is far from being true, there are still surprising results that are being uncovered. One such result has been obtained by the three PIs and their collaborators in 2019, it is the so-called `product structure theorem' for planar graphs. This theorem states that every planar graph is contained in the product of a simple tree-like graph and a path. This new theorem turned out to be a key tool allowing researchers to prove a number of old conjectures about planar graphs in the last three years.

The goal of this project is to find new applications of the product structure theorem for planar graphs, and study generalizations of the theorem to other classes of graphs. A number of related problems in structural graph theory will be studied as well.