Morse theoretical methods in Analysis, Dynamics, and Geometry.

Since the work of Morse on Riemannian geodesics, Morse theoretical methods have been powerful tools in the non-perturbative study of differential equations admitting a variational formulation. In its classical formulation, Morse theory is unfortunately not well-suited for the study of strongly indefinite functionals, namely of functionals whose critical points have infinite Morse index and co-index. A more modern approach, which allows to deal with the strongly indefinite case, has been introduced starting from the pioneering work of Smale in the 1960's and later of Floer in the 1980's, and has led to the development of Morse and Floer homology. Floer homology is since its development one of the most powerful tools in symplectic and contact geometry. However, with Floer homology it is usually very hard - due among other things to transversality issues - to obtain homotopy constructions.

Morse homology is instead more geometric in nature, its construction being based on the intersection of true geometric objects (the stable and unstable manifolds of pairs of critical points), and presents less transversality issues, as one has a larger class of admissible perturbations of a true (pseudo-)gradient vector field at disposal to achieve transversality. This suggests the possibility that the aforementioned homotopy constructions should be possible with Morse homology. However, this possibility has not been investigated yet.

On the other hand, the construction of Morse homology requires a good analytical setting. So far, Morse homology for strongly indefinite functionals has been constructed only on a Hilbert manifold setting, however in many applications the natural analytical setting is those of a Banach manifold. The aim of this project is to address these issues and can be summarized into the following two scientific aims:

i) develop a Morse homology theory for an abstract class of (possibly strongly indefinite) functionals on Banach manifolds as well as for specific classes of functionals, such as the one arising in the study of (perturbed) Dirac-harmonic maps, and

ii) push the study initiated with the previously funded NCN/ DFG-project "Morse theoretical methods in Hamiltonian dynamics" further, thus refining the variational methods alternative to Floer homology for the Hamiltonian action in cotangent bundles and toric manifolds.