BOOMER -
Boolean Methods for Expectations of Resolvents in Free Probability

In the modern world, understanding huge data sets is an important issue. A data set (table) from a mathematician's point of view is a matrix. One of the basic characteristics of a matrix is the so-called spectrum, which is the set of eigenvalues of the matrix. It turns out that in many cases for large random matrices their spectrum ceases to be random and behaves in a deterministic way.

So-called non-commutative probability provides a formalism that allows one to describe the asymptotic behavior of random matrices. It turns out that for independent random matrices that satisfy certain symmetry assumptions, the spectrum of the sum or product of matrices is described by the so-called free convolutions of the spectra of single matrices.

From an application point of view, it is important to have an in-depth understanding of the widest possible class of matrix transformations. The goal of this project is to create new and efficient tools which allow finding spectra for polynomials of matrices. Our project does not deal with the issue of convergence, which is a known fact. We will deal with the relevant operations at the level of non-commutative probability (i.e., after going with the matrix size "to infinity").

The above problem turns out to be much more complicated when the matrix which we obtain as a polynomial is not self-adjoint. The spectrum then is not contained in the set of real numbers. This situation is currently intensively studied and is much less understood than the self-adjoint case. Our aim is to propose a new approach also for the study of non-self-adjoint matrices.

To illustrate what we would like to understand, we present a graph of the eigenvalues of the product of certain random matrices of large size. We are interested in describing the limiting distribution of the spectrum in this and similar cases.