Abstract for the General Public

An instance of the Constraint Satisfaction Problem (CSP) consists of two parts. The one part defines variables which may, for example, correspond to tasks to be scheduled. The second part contains local restrictions on these variables, saying, for instance, that certain tasks have to take certain amount of time or have to be performed after other tasks are completed. The question in the CSP is whether there exists a global assignment to the variables, such that all the restrictions, called usually constraints, are satisfied. In our running example a satisfying assignment defines an order of performing the tasks.

Defining scheduling problems in the language of the Constraint Satisfaction Problem is beneficial from both practical and theoretical perspective. On the practical side, many common tools are designed to incorporate this formalism (take for example the algebraical scheduling language offered in IBM ILOG CP Optimizer, which is a component of IBM ILOG CPLEX Optimization Studio). Scheduling problems seen as CSPs involve e.g., projects in aerospace: project scheduling, aircraft assembly or airport scheduling: gates, landing, parking.

On the theoretical side, the formalism of CSPs generalizes the Boolean satisfiability problem. The Boolean satisfiability, in turn, defines the most important open question in theoretical computer science, a Millennium Prize Problem of the Clay Mathematics Institute: does NP equal P?

When constraints are modeled as relations over a finite domain, the CSP is always in NP. However, depending on the allowed set of relations, the CSP might be NP-complete (i.e. practically unsolvable by known algorithm) or in P (i.e. "efficiently" solvable). This "dichotomy" is obtained using, so called, algebraic approach to the CSP. The algebraic approach is a direction of research based on a deep connection between two involved mathematical theories: universal algebra and computational complexity.

Unfortunately, even the simplest scheduling problem requires considering relations over an infinite domain. The aim of this project is to make a next step beyond the finite-domain CSP and consider a larger class of problems, including some infinite-domain CSPs. The problems that can be modeled in such an extended framework involve scheduling, but also problems in spatial and temporal reasoning and many other areas. The class of problems is extended and thus the theories must follow suit: the new research builds on computational complexity, universal algebra, and this time model theory as well as Ramsey theory.