

Summary of the project for the general public.

The notion of limit is of fundamental importance in mathematics. Ancient Greek mathematicians were already aware of it – Archimedes, with an aid of so called infinitesimals, found a formula for the area of a circle and ellipse. In the 17th century, Isaac Newton and Gottfried Leibniz applied it to develop differential calculus. And around 200 years later, Bernhard Riemann defined integral, allowing (among many other things), to calculate areas of figures much more complicated than a circle.

Generally speaking, the concept of limit can be described as a method of approximating complex objects using simpler ones. Usually, limits are associated with numbers, however, it turns out that they arise in other contexts as well. In 1950-ties, Roland Fraïssé developed a theory allowing, e.g. to regard the ordering of rational numbers as a limit of finite linear orderings. Similarly, one can talk about limits of graphs, Banach spaces, Boolean algebras, etc.

Two crucial properties of Fraïssé limits are homogeneity and genericity. Roughly speaking, an object is homogeneous if none of its elements is distinguished from others. And it is generic if it is typical in some sense. To give an example, the Fraïssé limit of finite graphs is typical because, if we start constructing a countable graph in a random manner, e.g. by tossing a coin to decide whether there is an edge between two given vertices, with probability 1 we will obtain this particular limit graph.

In our project, we will study homogeneous and generic objects of various kinds, often resorting to Fraïssé limits. We will operate mainly in the areas of metric groups, dynamical systems, Banach spaces and operator algebras. We will try to find new examples of Fraïssé limits, investigate their properties, as well as extend their theory so that it can be applied in mathematically interesting situations that, as for now, go beyond it.